Important Concepts of Grade 6 Mathematics

W1 - Lesson 1 ............................................................. Basic Facts, Basic Operations, and Integers
W1 - Lesson 2 ............................................. Place Value, Whole Numbers, Decimals, and Common Fractions
W1 - Lesson 3 ....................................................... Improper Fractions and Mixed Numbers
W1 - Lesson 4 ................................................................... Ratios and Percents
W1 - Lesson 5 .................................................................. Number Operations with Decimals
W1 - Quiz
W2 - Lesson 1 ............................................................. Factors, Multiples, and Prime Factorizations
W2 - Lesson 2 ........................................................................ Metric Measurement
W2 - Lesson 3 ......................................................................... Perimeter and Area
W2 - Lesson 4 ........................................................................ Surface Area and Volume
W2 - Lesson 5 ............................................................. Working with Angles and Drawing Objects and Shapes
W2 - Quiz
W3 - Lesson 1 ........................................................................ Transformation
W3 - Lesson 2 ............................................................. Bar Graphs, Line Graphs, and Circle Graphs
W3 - Lesson 3 ........................................................................ Collecting and Analyzing Data
W3 - Lesson 4 ........................................................................ Number Patterns, Magic Squares, and Problem Solving
W3 - Lesson 5 ........................................................................ Probability and Outcomes
W3 - Quiz

Materials Required: A textbook is not needed. This is a stand-alone course.

Mathematics Grade 6
Version 5
Preview/Review W1 - Lesson 2 TEACHER KEY

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OBJECTIVES

By the end of this lesson, you should

- understand and use place value
- understand whole numbers
- understand that decimal fractions have values of less than one
- use correctly the signs for greater than, less than, and equals

GLOSSARY

decimal fractions: the numerals to the right of the decimal point with a value less than one

decimal point: in a number, the period used to separate the whole number from the fractional number with it
denominator: the bottom numeral of a fraction
equivalent fractions: two or more fractions that have the same value

\[
\begin{align*}
\text{e.g.,} & \quad \frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \\
\end{align*}
\]

numerator: the top numeral of a fraction

place value: the value of a digit determined by its position left or right of the decimal

proper fraction: a fraction in which the numerator is less than the denominator

\[
\begin{align*}
\text{e.g.,} & \quad \frac{2}{5} \\
\end{align*}
\]

whole numbers: numbers that are complete, without fractions
Welcome to W1 - Lesson 2! In this lesson you will examine place value with whole numbers and with numbers that are less than one. You will consider three topics:

- Place Value and Whole Numbers
- Place Value and Decimal Fractions
- Proper Fractions and Equivalent Fractions

You will write numbers in words, numbers in numerals, and numbers in expanded notation. You will review the signs for greater than, less than, and equals. You will create equivalent fractions by multiplying and dividing.

**Place Value and Whole Numbers**

You likely know that whole numbers are complete. That is, they do not have any fractions. *John has 2 apples. Sue has 14 jelly beans.*

However, if John ate half of one of his apples, how many apples would he have? You can write your answer to this in several ways.

- *John now has one and a half apples.*
- *John now has $1\frac{1}{2}$ apples.*
- *John now has 1.5 apples.*

A decimal point in a number means a fraction—a part of one in addition to the whole number. Therefore, John now has one and a part of another apple.
Numbers are made of digits. The value of the number is determined by the location of its digits. If we change the position of digits, we change the value of the number.

How do we read the number 123 456 789 012?

<table>
<thead>
<tr>
<th>Billions</th>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>100s</td>
<td>10s</td>
<td>1s</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Using the place value chart, you see that this number is made of

1 hundred billions
2 ten billions
3 billions
4 hundred millions
5 ten millions
6 millions
7 hundred thousands
8 ten thousands
9 thousands
0 hundreds
1 tens
2 ones

Therefore, 123 456 789 012 is read as one hundred twenty-three billion, four hundred fifty-six million, seven hundred eighty-nine thousand, twelve.

Notice that the number is grouped in sets of three. Each set of three digits in a number is a **triad**.
Questions

1. Rewrite the following numerals in words.

   **Example:** 74 826 = seventy-four thousand eight hundred twenty-six

   a. 4 566  *Four thousand five hundred sixty-six*

   b. 72 291  *Seventy-two thousand two hundred ninety-one*

   c. 845 032  *Eight hundred forty-five thousand thirty-two*

   d. 64 334 981  *Sixty-four million three hundred thirty-four thousand nine hundred eighty-one*

   e. 5 092 246  *Five million ninety-two thousand two hundred forty-six*

   f. 24 024 012 012  *Twenty-four billion twenty-four million twelve thousand twelve*
2. Below are some numbers in word form. Change them into numeral form (also referred to as standard form).

**Example:** eight hundred forty-nine thousand two hundred thirty-two = 849 232

a. seven hundred ninety-one

\[791\]

b. eighty-six thousand three hundred eleven

\[86 311\]

c. two hundred sixteen thousand four hundred twenty-five

\[216 425\]

d. three million nine thousand seven hundred

\[3 009 700\]

e. nine hundred thousand two hundred seventy-eight

\[900 278\]

f. twenty-two million five hundred six thousand thirty-one

\[22 506 031\]

g. seventeen million sixteen thousand and ten

\[17 016 010\]

h. sixty-five billion four hundred nine thousand eighty-eight

\[65 000 409 088\]
3. Write the value of the digit shown in **bold** type.

**Examples:**  123 456 789 = 4 hundred thousands; 67 903 = 9 hundreds

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td><strong>56 254</strong></td>
<td><strong>6 thousands</strong></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td><strong>2 445 771</strong></td>
<td><strong>2 millions</strong></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td><strong>348 995</strong></td>
<td><strong>9 hundreds</strong></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td><strong>2 034</strong></td>
<td><strong>3 tens</strong></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td><strong>879 547</strong></td>
<td><strong>8 hundred thousands</strong></td>
<td></td>
</tr>
<tr>
<td>f.</td>
<td><strong>234 764 560</strong></td>
<td><strong>3 ten millions</strong></td>
<td></td>
</tr>
<tr>
<td>g.</td>
<td><strong>4 212 659 559</strong></td>
<td><strong>2 hundred millions</strong></td>
<td></td>
</tr>
<tr>
<td>h.</td>
<td><strong>58 905</strong></td>
<td><strong>5 ones</strong></td>
<td></td>
</tr>
<tr>
<td>i.</td>
<td><strong>340 662</strong></td>
<td><strong>4 ten thousands</strong></td>
<td></td>
</tr>
<tr>
<td>j.</td>
<td><strong>984 707 033</strong></td>
<td><strong>4 millions</strong></td>
<td></td>
</tr>
</tbody>
</table>
4. We sometimes write in numerals the place values we speak as words. Write the following numbers in the **expanded notation** form.

**Examples:**

579 135 = 500 000 + 70 000 + 9 000 + 100 + 30 + 5
20 508 604 = 20 000 000 + 500 000 + 8 000 + 600 + 4

a. 1 357 = \(1000 + 300 + 50 + 7\)

b. 3 905 = \(3000 + 900 + 5\)

c. 677 321 = \(600 000 + 70 000 + 7 000 + 300 + 20 + 1\)

d. 26 789 343 = \(20 000 000 + 6 000 000 + 700 000 + 80 000 + 9 000 + 300 + 40 + 3\)

e. 99 876 322 = \(90 000 000 + 9 000 000 + 800 000 + 70 000 + 6 000 + 300 + 20 + 2\)

f. 451 672 893 014 = \(400 000 000 000 + 50 000 000 000 + 1 000 000 000 + 600 000 000 + 70 000 000 + 2 000 000 + 800 000 + 90 000 + 3 000 + 10 + 4\)
5. Write the numbers represented by these place-value statements.

**Example:** 7 hundred thousands + 6 thousands + 9 hundreds + 2 tens = 706 920

a. 3 thousands + 5 hundreds + 6 tens + 3 ones =
   
   3 563

b. 9 ten thousands + 7 thousands + 2 hundreds + 2 ones =
   
   97 202

c. 3 ten millions + 9 millions + 3 hundred thousands + 6 thousands + 7 hundreds + 4 tens =
   
   39 306 740

d. 9 hundred millions + 5 hundred thousands + 2 ten thousands + 9 tens =
   
   900 520 090

e. 3 ten millions + 8 ten thousands + 9 hundreds + 6 ones =
   
   30 080 906

f. 4 hundred billions + 1 billion + 9 ten millions + 2 hundred thousands + 4 ten thousands + 9 thousands + 6 hundreds + 4 ones =
   
   401 090 249 604

987 = nine hundred +
Mathematicians use many symbols! You are familiar with + for plus or add, - for subtract, × for multiply, = for equals, and so on.

Perhaps you remember that > means greater than and < means less than. However, do you sometimes forget which way to draw the sign? Try to remember that it is an arrowhead that points to the smaller one.

For example, 6 is greater than 4, or 6 > 4. Because 4 is less than 6, we write 4 < 6, and we say 4 is less than 6. (Perhaps you can hear 6 saying, “Hey, little guy! I’m bigger than you!” Sometimes he points the other way, but large always points at small.)

6. Insert the correct sign to make the following statements true (= or > or <) (Examples: 22 > 10, 16 < 100 and 33 = 33)

   a. 109 ______<_______ 901
   b. 7 666 ______>_______ 6 777
   c. 333 220 ______>_______ 333 022
   d. 987 789 987 ______=_______ 987 789 987
   e. 876 543 345 ______<_______ 876 543 354
   f. 620 620 123 456 ______>_______ 620 602 123 456
7. Construct numbers by following the instructions. Underline the requested digits in your answer.

**Example**: Write a five-digit number with a six in the tens place. Use the 6 only once. Answers may vary: 23 468 is one example.

a. Write a five-digit number with a 7 in the ten thousands location and a 3 in the hundreds location. Use the 7 and 3 only once.

  **Answers will vary except for underlined digits 71 323**

b. Write an eight-digit number with a 2 in the ten millions location and a 4 in the tens location. Use the 4 and 2 only once.

  **Example** 21 356 748 (Answers will vary.)

c. Write a seven-digit number with a 9 in the millions location and a 1 in the ten thousands location. Use the 9 and 1 only once.

  **Example** 9 213 456 (Answers will vary.)

d. Use the following digits (1, 3, 5, 7, 9, 2, 4, 6, 0) to write the largest nine-digit number possible. Use each digit only once.

  **976 543 210** (This answer should not vary.)

e. Start with 998 877 665 and create a new number by subtracting eight thousand.

  **998 869 665**
Place Value and Decimal Numbers

Place Value Chart

When we read and write decimal fractions, the words always end in -ths.

- $0.4 = four \text{ tenths}$
- $0.17 = seventeen \text{ hundredths}$
- $0.234 = two \text{ hundred thirty-four thousandths}$

How do you read the number 456.789?

<table>
<thead>
<tr>
<th>100s</th>
<th>10s</th>
<th>1s [decimal]</th>
<th>10ths</th>
<th>100ths</th>
<th>1 000ths</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>.</td>
<td>7</td>
<td>8 9</td>
</tr>
</tbody>
</table>

Using the place value chart you see that this number is made of

- 4 hundreds
- 5 tens
- 6 ones
- 7 tenths
- 8 hundredths
- 9 thousandths

Therefore, 456.789 is read as four hundred fifty-six and seven hundred and eighty nine thousandths.
Questions

1. Rewrite the following numerals in words.

   **Example:** 20.45 = twenty and forty-five hundredths

   a. 8.6 = **eight and six tenths**
   b. 3.19 = **three and nineteen hundredths**
   c. 0.254 = **two hundred fifty-four thousandths**
   d. 2.07 = **two and seven hundredths**
   e. 9.306 = **nine and three hundred six thousandths**

2. Below are some numbers in word form. Change them into numeral form.

   **Example:** two hundred ten and three hundred fifteen thousandths = 210.315

   a. one and seven tenths = **1.7**
   b. three and twenty-four hundredths = **3.24**
   c. six and six hundred three thousandths = **6.603**
   d. nine and five hundredths = **9.05**
   e. eight and twenty-two thousandths = **8.022**
   f. sixty-six and eight thousandths = **66.008**
3. Write the value of the digit shown in **bold** type.

**Example:** \(89.683 = \text{eight hundredths}\)

a. \(7.86 = \underline{8 \text{ tenths}}\)

b. \(2.654 = \underline{5 \text{ hundredths}}\)

c. \(9.071 = \underline{1 \text{ thousandths}}\)

d. \(43.85 = \underline{3 \text{ ones}}\)

e. \(67.808 = \underline{8 \text{ thousandths}}\)

f. \(0.454 = \underline{4 \text{ tenths}}\)

g. \(33.163 = \underline{6 \text{ hundredths}}\)

4. Write the following numbers in the expanded notation form.

**Example:** \(18.286 = 10 + 8 + \frac{2}{10} + \frac{8}{100} + \frac{6}{1000}\)

a. \(7.8 = \underline{7 + \frac{8}{10}}\)

b. \(9.56 = \underline{9 + \frac{5}{10} + \frac{6}{100}}\)

c. \(28.386 = \underline{20 + 8 + \frac{3}{10} + \frac{8}{100} + \frac{6}{1000}}\)

d. \(3.047 = \underline{3 + \frac{4}{100} + \frac{7}{1000}}\)

e. \(345.507 = \underline{300 + 40 + 5 + \frac{5}{10} + \frac{7}{1000}}\)
5. Write the numbers represented by these place-value statements.

Example: 4 ones + 3 tenths + 6 hundredths = 4.36
a. 7 ones + 8 tenths + 4 hundredths = 7.84
b. 8 ones + 5 hundredths + 2 thousandths = 8.052
c. 4 tens + 16 hundredths = 40.16
d. 8 hundreds + thirty-six thousandths = 800.036
e. 5 thousands + one hundred seven thousandths = 5000.107

6. Insert the correct sign to make the following statements true (= or > or <).

a. 25.6 __ > __ 2.56
b. 78.45 __< __ 78.45
c. 45.90 __ = __ 45.9
d. 23.099 __ < __ 23.11
e. 122.4 __ > __ 122.334
f. 4.7 __ > __ 0.998

7. Construct numbers by following the instructions below. Underline the designated digits in your answers.

Example: Write a four-digit number with a 6 in the ones place and a 5 in the hundredths place. Use the 6 and 5 only once. Answer may vary. One answer is 86.25.

Answers may vary

a. Write a four-digit number with a 9 in the tens place and the hundredths place. Use the 9 only twice. ________ 91.09
b. Write a four-digit number smaller than 1 with a 3 in the thousandths place. Use the 3 only once. ________ 0.003
c. Write a seven-digit number with a 2 in the thousands place and the thousandths place. Use the 2 only twice.  

\[2000.002\]

**Proper Fractions and Equivalent Fractions**

The **numerator** is the top numeral of a fraction.

The **denominator** - the bottom number of a fraction.

A **proper fraction** is a fraction in which the numerator is less than the denominator. The value of a proper fraction is always less than one.

\[\frac{2}{5}\] (numerator)

\[\frac{5}{5}\] (denominator)

**Equivalent Fractions** are two or more fractions that have the same value.

\[\frac{1}{2}, \frac{2}{4}, \frac{3}{6}\] are equivalent fractions.

**Making Equivalent Fractions**

A fraction is changed into an equivalent fraction by **multiplying** both the numerator and denominator of the fraction by the same number. Any multiplier may be used.

\[\frac{\text{Numerator} \times n}{\text{Denominator} \times n} = \text{Equivalent Fraction}\]

\[\frac{1}{2} \times 2 = \frac{2}{4}\] (\(\frac{1}{2}\) and \(\frac{2}{4}\) are equivalent fractions)

\[\frac{2}{8} \div 2 = \frac{1}{4}\]
Equivalent fractions are made by dividing both the numerator and denominator by the same number.

\[
\frac{6}{24} \div \frac{3}{3} = \frac{2}{8} \quad \text{and} \quad \frac{6}{24} \quad \text{are equivalent fractions}
\]

So, \(\frac{6}{24}\), \(\frac{2}{8}\), and \(\frac{1}{4}\) are all equivalent fractions.

Questions
1. Rewrite the following numerals in words.

   Example: \(\frac{3}{5} = \text{three fifths}\) \(\frac{17}{35} = \text{seventeen thirty-fifths}\)

   a. \(\frac{1}{4} = \text{one fourth or one quarter}\)

   b. \(\frac{73}{100} = \text{seventy-three hundredths}\)

   c. \(\frac{5}{8} = \text{five eighths}\)

   d. \(\frac{41}{50} = \text{forty-one fiftieths}\)

   e. \(\frac{779}{1000} = \text{seven hundred seventy-nine thousandths}\)
2. Rewrite the following numbers as proper fractions.

**Example:** four tenths = \( \frac{4}{10} \)

a. six hundredths = \( \frac{6}{100} \)

b. eleven hundredths = \( \frac{11}{100} \)

c. two hundred and twenty-two thousandths = \( \frac{222}{1000} \)

d. three twentieths = \( \frac{3}{20} \)

e. eighty-one two hundredths = \( \frac{81}{200} \)

3. Make three equivalent fractions for each of the following fractions by multiplying. Choose your own multipliers.

**Example:** \( \frac{1}{2} = \frac{2}{4}, \frac{3}{6}, \frac{4}{8} \) The multipliers used were 2, 3, and 4.

*Answers will vary*

a. \( \frac{4}{7} = \frac{8}{14}, \frac{12}{21}, \frac{16}{28} \)

b. \( \frac{7}{10} = \frac{14}{20}, \frac{21}{30}, \frac{28}{40} \)

c. \( \frac{9}{25} = \frac{18}{50}, \frac{27}{75}, \frac{36}{100} \)

d. \( \frac{22}{100} = \frac{44}{200}, \frac{66}{300}, \frac{88}{400} \)

e. \( \frac{333}{1000} = \frac{666}{2000}, \frac{999}{3000}, \frac{1332}{4000} \)
4. Make three equivalent fractions for each of the following fractions by dividing. Choose your own divisors.

**Example:** \( \frac{50}{100} = \frac{25}{50}, \frac{5}{10}, \frac{1}{2} \). The divisors used were 2, 10, and 50.

*Answers will vary*

a. \( \frac{150}{200} = \frac{75}{100}, \frac{15}{20}, \frac{30}{40} \)

b. \( \frac{100}{1000} = \frac{50}{100}, \frac{10}{100}, \frac{1}{10} \)

c. \( \frac{60}{80} = \frac{30}{40}, \frac{12}{16}, \frac{6}{8} \)

d. \( \frac{100}{150} = \frac{50}{75}, \frac{20}{30}, \frac{2}{3} \)

e. \( \frac{48}{60} = \frac{24}{30}, \frac{12}{15}, \frac{4}{5} \)

5. Insert the correct sign to make the following statements true (= or > or <):

a. \( \frac{34}{100} < \frac{87}{100} \)

b. \( \frac{66}{100} > \frac{66}{1000} \)

c. \( \frac{2}{10} > \frac{2}{100} \)

d. \( \frac{11}{1000} < \frac{111}{100} \)
6. Determine whether the following pairs of fractions are equivalent. Write equivalent or not equivalent in the answer spaces.

Examples: \( \frac{3}{5} \), \( \frac{6}{10} \) equivalent; \( \frac{3}{5} \), \( \frac{3}{10} \) not equivalent

a. \( \frac{2}{3} \), \( \frac{10}{15} \) equivalent

b. \( \frac{3}{4} \), \( \frac{9}{12} \) equivalent

c. \( \frac{25}{30} \), \( \frac{25}{60} \) not equivalent

d. \( \frac{2}{5} \), \( \frac{30}{100} \) not equivalent

e. \( \frac{24}{32} \), \( \frac{3}{4} \) equivalent

Homework Assignment

Use the following numbers to answer the questions below.

\[
\begin{align*}
325.615 & \quad 2399.994.821 & \quad 404.715,666.102 & \quad 4286.4 \\
278.61 & \quad 151.926 & \quad 6401.235.57 & \\
2488.4 & \quad 78845.789.3 & \quad 691.461,287.251 & \\
\end{align*}
\]
1. Which number is the largest?

691 461 287.251

2. Which number is the smallest?

151.926

3. Which number has the smallest digit in the tenths place value location?

404 715 666.102

4. Which number has the largest digit in the hundred thousands place value location?

78 845 789.3

5. Which number has a 9 in the ten millions place value location?

691 461 287.251

6. Write the three largest numbers, and then calculate their sum.

691 461 287.251 + 404 715 666.102 + 78 845 789.3 =

1 175 022 742.653

7. Write the largest and smallest numbers and then calculate their difference.

691 461 287.251 - 151.926 = 691 461 135.325
Self-Evaluation

Ask yourself some important questions. Write your answers in sentences for your teacher.

1. In this lesson, what part of your work was excellent?

2. In this lesson, what part of your work needs improvement?

3. If you want help for some of the work in this lesson, ask your teacher in this space.