Important Concepts . . .

Preview Review

Mathematics Grade 6 TEACHER KEY

W2 - Lesson 1: Factors, Multiples, and Prime Factorizations

V5-07

Important Concepts of Grade 6 Mathematics

Materials Required: A textbook is not needed. This is a stand-alone course.

Mathematics Grade 6
Version 5
Preview/Review W2 - Lesson 1 TEACHER KEY

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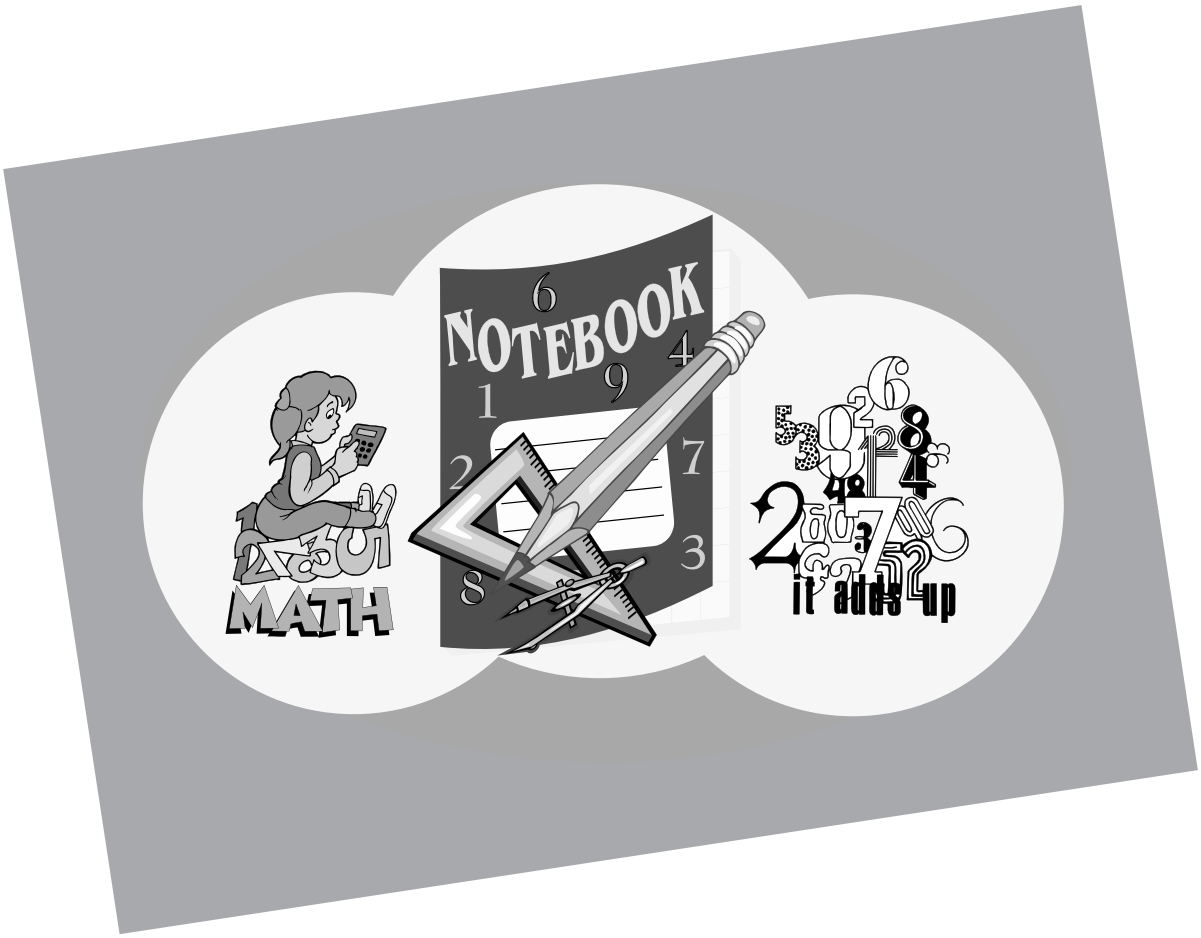
Preview/Review Concepts for Grade Six Mathematics

TEACHER KEY



W2 - Lesson 1: Factors, Multiples, and Prime Factorizations

OBJECTIVES

By the end of this lesson, you should

- find the least common multiple of given numbers
- find the greatest common factor of given numbers
- use a factor tree to find the prime factors of a given number

GLOSSARY

common factors - factors shared by different numbers

common multiple - a multiple shared by given numbers

composite number - a number with three or more factors

factors - numbers used to form a product

greatest common factor (GCF)- the factor that is greatest among all the common factors of given products

least common multiple (LCM) - the multiple that is least (or lowest) among all the common multiples of given numbers

multiple - the product of a given number and a whole number greater than zero

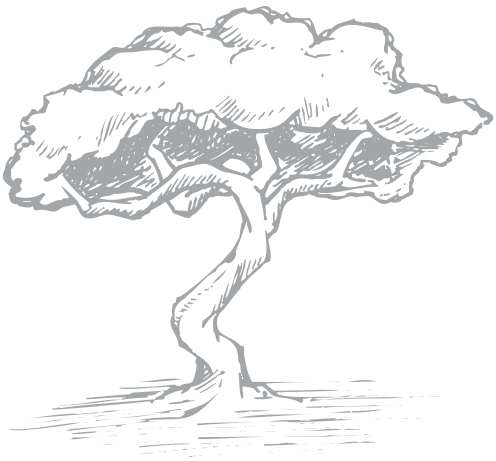
prime factorization - a process of writing a number as a product of prime numbers

prime number - a number whose only factors are one and itself

W2 - Lesson 1: Factors, Multiples, and Prime Factorization

Welcome to W2 - Lesson 1! This lesson is about multiples and factors. You will learn to find and use least common multiples and greatest common factors. The lesson has three topics:

- Multiples, Common Multiples, and Least Common Multiples
- Prime and Composite Numbers
- Common Factors, Greatest Common Factors, and Prime Factorization



You will also use factor trees to add to your skills in using numbers.

Multiples, Common Multiples, and Least Common Multiples

Because you have been multiplying numbers, you likely know that a multiple is the product of a given number and a whole number greater than zero. For example, some multiples of 7 are 7, 14, 21, 28, and 35. You can find these by multiplying 7 by 1, then by 2, and so on.

A common multiple is a multiple shared by given numbers. For example, 12 is a multiple of 2, 3, and 4, so 12 is a common multiple of 2, 3, and 4.

$2 \times 1 = 2$ $2 \times 2 = 4$ $2 \times 3 = 6$ $2 \times 4 = 8$ $2 \times 5 = 10$ $2 \times 6 = \boxed{\mathbf{12}}$

$3 \times 1 = 3$ $3 \times 2 = 6$ $3 \times 3 = 9$ $3 \times 4 = \boxed{\mathbf{12}}$

$4 \times 1 = 4$ $4 \times 2 = 8$ $4 \times 3 = \boxed{\mathbf{12}}$

The least common multiple (LCM) is the multiple that is the least or lowest among all the common multiples of given numbers.

Multiples of 4 are 4, 8, **12**, 16, 20, 24 …
Multiples of 6 are 6, **12**, 18, 24, 30 …
The LCM of 4 and 6 is 12.

Questions

1. Write 5 more multiples of each of the following numbers.

   Example: 4: 8, 12, 16, 20, 24

   a. 3: ***6, 9, 12, 15, 18***

   b. 5: ***10, 15, 20, 25, 30***

   c. 8: ***16, 24, 32, 40, 48***

   d. 15: ***30, 45, 60, 75, 90***

   e. 32: ***64, 96, 128, 160, 192***

2. For each of the following pairs of numbers, write the first <u>three</u> common multiples.

   Example: Multiples of 2: 2, 4, 6, **8**, 10, 12, 14, **16**, 18, 20, 22, **24**, 26 … Multiples of 8: **8**, **16**, **24**, 32, 40 … Common multiples of 2 and 8 are **8**, **16** and **24**.

   a. 4: ***4, 8, 12, 16, 20, 24***

      8: ***8, 16, 24, 32***

      The first three common multiples are ***8, 16, 24***

   b. 2: ***4, 6, 8, 10, 12, 14, 16, 18, 20, 22,24, 26, 28, 30***

      5: ***10, 15, 20, 25, 30, 35***

      The first three common multiples are ***10, 20, 30***

c. 4: ***8, 12, 16, 20, 24, 28, 32, 36***

6: ***12, 18, 24, 30, 36***

The first three common multiples are ***12, 24, 36***

d. 6: ***12, 18, 26, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90***

10: ***20, 30, 40, 50, 60, 70, 80, 90***

The first three common multiples are ***30, 60, 90***

e. 3: ***6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48***

8: ***16, 24, 32, 40, 48, 56, 64, 72, 80***

The first three common multiples are ***24, 48, 72***

3. For each group of three numbers, write the Least Common Multiple:

Example: Multiples of 2: 4, 6, 8, 10, **12**, 14, 16 …
Multiples of 4: 8, **12**, 16, 20 …
Multiples of 6: **12**, 18, 24, 30 …
Least Common Multiple (LCM) of 2, 4 and 6 is 12.

a. 3: ***6, 9, 12, 15, 18***

6: ***12, 18***

9: ***18***

The LCM is ***18***

b. 4: ***8, 12, 16, 20, 24***

6: ***12, 18, 24***

8: ***16, 24***

The LCM is ***24***

c. 2: ***4, 6, 8, 10***

5: ***10***

10: ***10***

The LCM is ***10***

d. 3: ***6, 9, 12, 15, 18, 21***

7: ***14, 21***

21: ***21***

The LCM is ***21***

4. George takes a trip to Australia every two years. Manuel travels to Australia every three years. Jonathon visits Australia every four years. All three men took a trip to Australia in the year 2000. In which year will all three men visit Australia again on the same year?

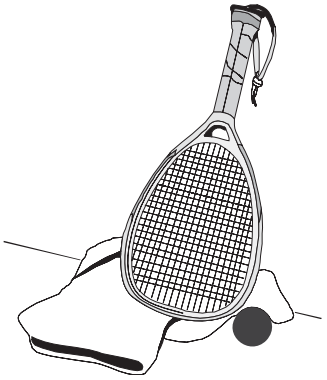
George → 2 years → 2002, 2004, 2006, 2008, 2010, 2012

Manuel → 3 years → 2003, 2006, 2009, 2012

Jonathon → 4 years → 2004, 2008, 2012

**** In the year of 2012, all three will visit Australia again.***

5. Pam plays racquetball on every third day of the week. Monique does weight training every second day. Mary runs laps every fifth day. Pam, Monique and Mary met at the Fitness Gymnasium on Friday, July 4th. If they follow their routines, in how many days from July 4th will they meet at the Fitness Gymnasium again?



Pam → third day → July 7, 10, 13, 16, 19, 22, 25, 28, 31, Aug. 3

Monique → second day → July 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, Aug. 1, 3

Mary → fifth day → July 9, 14, 19, 24, 29, Aug. 3

They will meet again in 30 days on August 3.

Prime and Composite Numbers

You will recall that **factors** are numbers used to form a product. For example, 2 and 6 are factors of 12, and 4 and 5 are factors of 20.

Another term you have used before is **prime number,** a number whose only factors are 1 and itself. For example, the factors of 5 are 1 and 5.

A number with three or more factors is a **composite number.** For example, the factors of 10 are 1, 10, 2, and 5. Therefore, 10 is a composite number.

> ***Remember**: 1 is not a prime number or a composite number. It has only one factor. 1× 1 = 1*

Do you know the trick to find if a number is divisible by three? Add the digits in the number together. If the sum of the digits is divisible by 3, then the whole number is divisible by 3.

> **27 = 2 + 7 = 9**
> 9 is divisible by 3 so 27 is also divisible by 3.
>
> **126 = 1 + 2 + 6 = 9**
> 9 is divisible by 3 so 126 is also divisible by 3.
>
> **4 578 = 4 + 5 + 7 + 8 = 24**
> 24 is divisible by 3 so 4 578 is also divisible by 3.
>
> **24 = 2 + 4 = 6**
> 6 is divisible by 3 so 24 is also divisible by 3.
>
> ***Remember**: If a number is divisible by three, it is a **composite number**. This is true for all numbers divisible by 3, except for the number 3.*

Questions

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Check chart for appropriate shading (as below)

1. Use the chart above to find the prime numbers from 1 to 100. Shade all the numbers that are **not** prime numbers.

   a. 1 is not a prime number. Shade in the 1 on the chart.

   b. 2 is a prime number. It has two factors: 1 and 2. Do **not** shade in the 2. All other even numbers are composite numbers. Write all the multiples of 2 up to 100 on the lines below and then shade them in on the chart, then shade in all the even numbers (except 2) on the chart.

   2, 4, 6, 8, 10, 12, 14, 16, 18, ... up to 100 (by two's)

c. 3 is a prime number.
It has two factors: 1 and 3.
All other multiples of 3 are composite numbers.
Write all the multiples of 3 up to 100 on the lines below and shade all the multiples of 3 (except 3) on the chart.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ... to 100 (by three's)

d. 5 is a prime number.
It has two factors: 1 and 5.
All other multiples of 5 are composite numbers.
Write the multiples of 5 up to 100 on the lines below and shade all the multiples of 5 (except 5) on the chart.

5, 10, 15, 20, 25, 30, ... to 100 (by five's)

e. 7 is a prime number.
It has two factors: 1 and 7.
All other multiples of 7 are composite numbers.
Write the multiples of 7 up to 100 on the lines below and shade all the multiples of 7 (except 7) on the chart.

7, 14, 21, 28, 35, 42, 49, 56, ... to 100 (by seven's)

2. There are 25 prime numbers between 1 and 100. List them below.

2, 3, 5, 7, 11, 13, 17, 19, 23, 31, 37, 41, 43, 47, 53, 59, 61, 71,

79, 83, 89, 97

3. Using the rule listed earlier in this lesson, decide if the following numbers are divisible by three. Write **yes** or **no**.

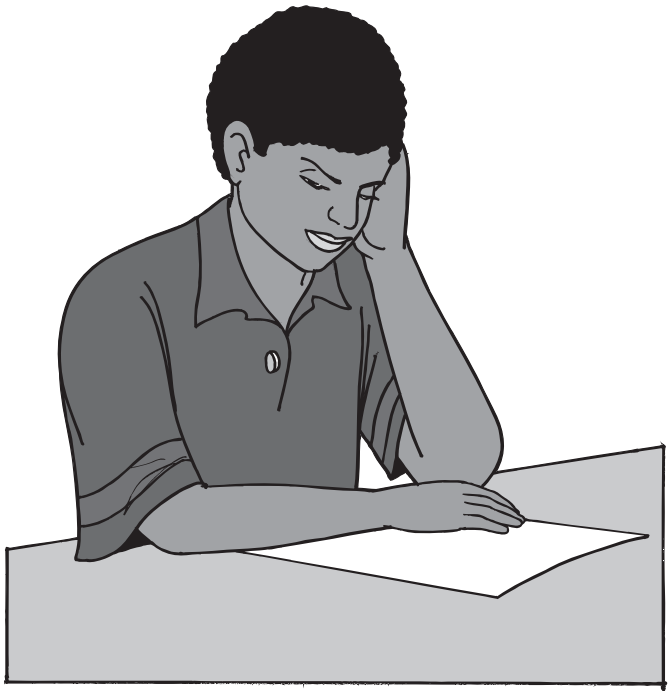
a. 27 ***yes*** b. 66 ***yes***

c. 100 ***no*** d. 123 ***yes***

e. 124 ***no*** f. 789 ***yes***

g. 23 231 ***no*** h. 88 662 ***yes***

i. 88 669 ***no*** j. 101 010 ***yes***



4. Beside each number write **P** for **prime** or **C** for **composite**.

a. 20	***C***	b. 35	***C***
c. 333	***C***	d. 83	***P***
e. 89	***P***	f. 101	***P***
g. 102	***C***	h. 111	***C***
i. 201	***C***	j. 203	***C***

5. Circle the two prime numbers in the list below.

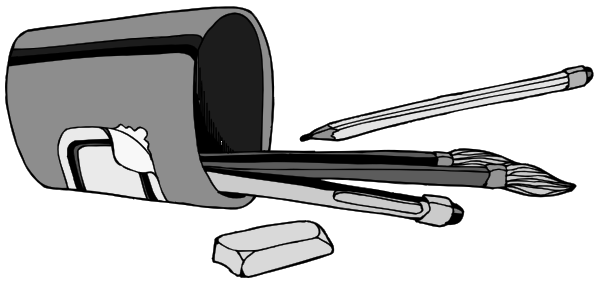
(151), 152, 153, 154, 155, 156, (157), 158, 159, 160.

6. List the three prime numbers between 260 and 275.

263, 271, 273

7. Write the four prime numbers between 900 and 915.

901, 907, 911, 913



Common Factors, Greatest Common Factors and Prime Factorization

Common Factors are factors that are shared by different products. For example, 4 is a factor of 12 and of 20; therefore, 4 is a common factor of 12 and 20.

When we consider all the factors of two numbers, we can find the **greatest common factor** (GCF), the factor that is the greatest among all the common factors of different products.

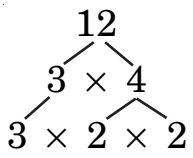
Factors of 8: (1, 8, 2, **4**)
Factors of 12: (1, 12, 2, 6, 3, **4**)
Factors of 20: (1, 20, 2, 10, **4**, 5)
Answer: The GCF of 8, 12 and 20 is **4**.

You can use prime numbers in **prime factorization.** This is a process of writing a number as a product of prime numbers.

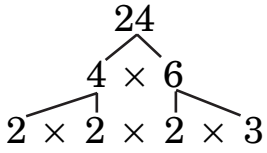
$25 = 5 \times 5$

$36 = 2 \times 3 \times 2 \times 3$

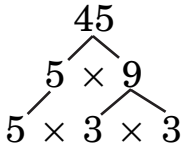
You can build a **factor tree** by writing the number on the top line. On the second line, give two factors. Continue to find factors of the number until all of the numbers on the bottom line are prime numbers.



4 is not a prime number.
The bottom line has prime numbers.



4 and 6 are not prime numbers.
The bottom line has prime numbers.



9 is not a prime number.
The bottom line has prime numbers.

Questions

1. Write all of the factors of the following numbers:

   Example: 15 = 1, 15, 3, 5 The factors of 15 are 1, 3, 5, and 15

   a. 8 = ***1, 8, 2, 4***

   b. 12 = ***1, 12, 2, 6, 3, 4***

   c. 32 = ***1, 32, 2, 16, 4, 8,***

   d. 50 = ***1, 50, 2, 25, 5, 10,***

   e. 84 = ***1, 84, 2, 42, 3, 28, 4, 21, 6, 14, 7, 12***

2. For each pair of numbers, write all of the factors of each number and then write the greatest common factor (GCF).

   Example: 10 = 1, 10, 2, 5 The factors of 10 are 1, 2, 5, and 10.
   15 = 1, 15, 3, 5 The factors of 15 are 1, 3, 5, and 15. The GCF is 5.

   a. 7 = ***1, 7***

      14 = ***1, 14, 2, 7***

      The GCF is ***7***

   b. 12 = ***1, 12, 2, 6, 3, 4***

      16 = ***1, 16, 4, 2, 8***

      The GCF is ***4***

c. 22 = ***1, 22, 2, 11***

33 = ***1, 33, 3, 11***

The GCF is ***11***

d. 17 = ***1, 17***

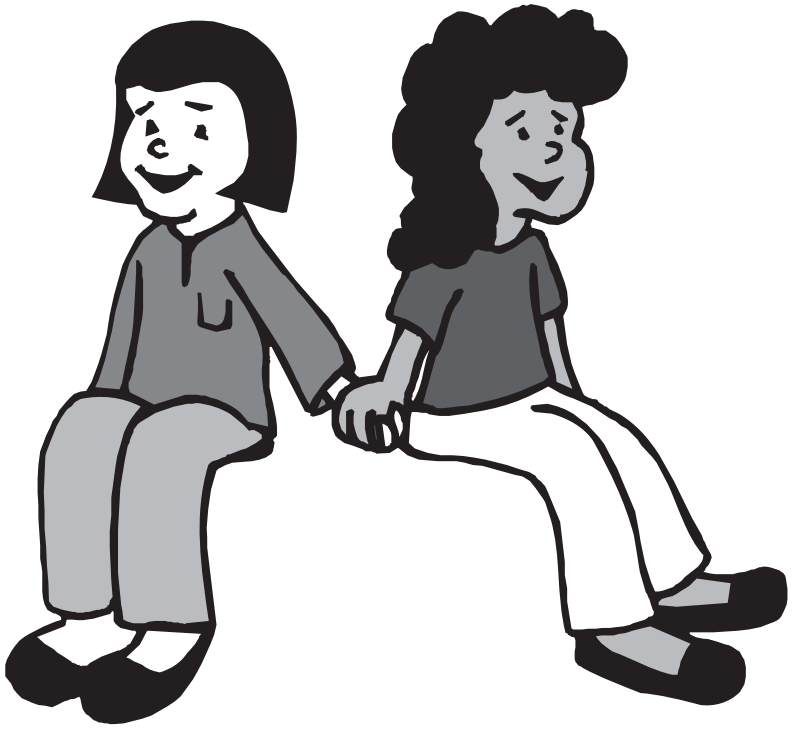
34 = ***1, 34, 2, 17***

The GCF is ***17***

e. 24 = ***1, 24, 2, 12, 3, 8, 4, 6***

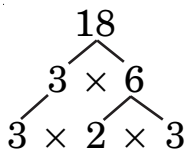
64 = ***1, 64, 2, 32, 4, 16, 8***

The GCF is ***8***



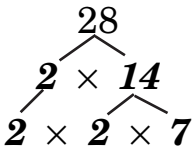
3. Make a factor tree for each of the following numbers. Use the space below to draw your factor trees.

Example:

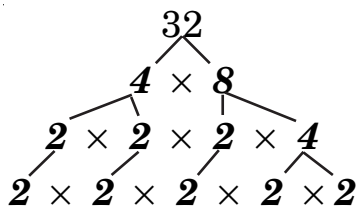


The bottom line are all prime numbers.

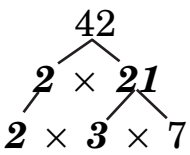
a.



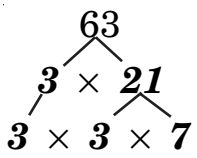
b.



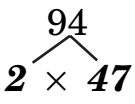
c.



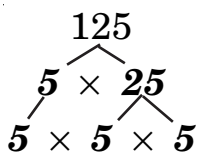
d.



e.



f.



4. Below are several fractions. By dividing the numerator and denominator of each fraction by the same whole number, we can make lowest-terms fractions.

   Note: We can use GCF to help us find the lowest-terms fraction.

   Example: Write $\frac{15}{75}$ as a lowest-term fraction.

   First: Write the factors of 15 and 75 and then find the GCF.
   15 = 1, 15, 3, 5 The factors of 15 are 1, 3, 5 and **15**
   75 = 1, 75, 3, 25, 5, 15 The factors of 75 are 1, 3, 5, **15**, 25 and 75
   The GCF is **15**.
   Second: Divide both the numerator and denominator by the GCF.
   15 ¸ 15 = 1
   75 ¸ 15 = 5

   The lowest-terms fraction of $\frac{15}{75} = \frac{1}{5}$.

   a. $\frac{6}{8}$ = ***factors of 6 = 1, 6, 2, 3***

   factors of 8 = 1, 8, 2, 4

   GCF = ***2***

   The lowest-terms fraction is $\frac{3}{4}$

   b. $\frac{18}{45}$ = ***factors of 18 = 1, 18, 2, 9,3, 6***

   factors of 45 = 1, 45, 3, 15, 5, 9

   GCF = ***9***

   The lowest-terms fraction is $\frac{2}{5}$

c. $\frac{9}{39}$ = ***factors of 9 = 1, 9, 3***

factors of 39 = 1, 39, 3, 13

GCF = ***3***

The lowest-terms fraction is $\frac{3}{13}$

d. $\frac{92}{104}$ = ***factors of 92 = 1, 92, 2, 46, 4, 23***

factors of 104 = 1, 104, 2, 52, 4, 26, 8, 13

GCF = ***4***

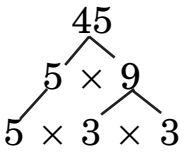
The lowest-terms fraction is $\frac{23}{26}$

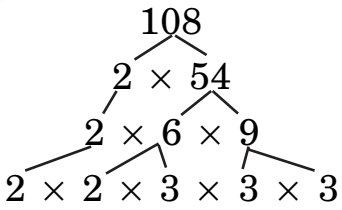
5. Below are some fractions. Create an equivalent fraction by dividing the numerator and denominator by the same number.

Note: We can use the prime factors of given numbers when we are trying to make equivalent fractions by division.

Example: Find an equivalent fraction of $\frac{45}{108}$ by dividing.

First: Draw a factor tree for each number.





The prime factors of 45 are $3 \times 3 \times 5$.
The prime factors of 108 are $2 \times 2 \times 3 \times 3 \times 3$.

Second: Divide the numerator and denominator of the fraction 45/108 by a common prime number. Both 45 and 108 have 3 as a prime factor. So we can create an equivalent fraction by dividing by 3.

$$\frac{45 \div 3}{108 \div 3} = \frac{15}{36}$$

$\frac{45}{108}$ and $\frac{15}{36}$ are equivalent fractions.

a. $\frac{25}{95}$ = ***Prime Factors of*** $\begin{array}{c} \boldsymbol{25} \\ \diagup\ \diagdown \\ \boldsymbol{5 \times 5} \end{array}$

Factor trees: ***Prime Factors of*** $\begin{array}{c} \boldsymbol{95} \\ \diagup\ \diagdown \\ \boldsymbol{5 \times 19} \end{array}$

Common prime number is ***5***

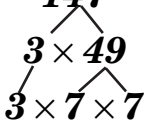
The equivalent fraction is $\boldsymbol{\frac{25 \div 5}{95 \div 5} = \frac{5}{19}}$

b. $\frac{21}{147} =$

Factor trees:

Prime Factors of 21

Prime Factors of 147



Common prime number is ***3 or 7***

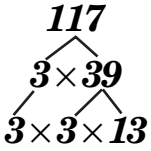
The equivalent fraction is $\frac{7}{49}$ ***or*** $\frac{3}{21}$ ***or*** $\frac{1}{7}$

c. $\frac{13}{117} =$

Factor trees:

Prime Factors of 13

Prime Factors of 117



Common prime number is ***13***

The equivalent fraction is $\frac{13 \div 13}{117 \div 13} = \frac{1}{9}$

Homework Assignment

1. List all the factors of each group of numbers and find the greatest common factor (GCF).

   a. 10 = ***1, 10, 2, 5***

   15 = ***1, 15, 3, 5***

   GCF is ***5***

   b. 6 = ***1, 6, 2, 3***

   24 = ***1, 24, 2, 12, 6, 4, 3, 8***

   30 = ***1, 30, 2, 15, 3, 10, 5, 6***

   GCF is ***6***

   c. 14 = ***1, 14, 2, 7***

   28 = ***1, 28, 2, 14, 4, 7***

   42 = ***1, 42, 2, 21, 3, 14, 6, 7***

   GCF is ***14***

2. Draw a factor tree for each of the following numbers.

   a. 99
   3 × 33
   3 × 3 × 11

   b. 175
   5 × 35
   5 × 5 × 7

   c. 246
   2 × 123
   2 × 3 × 41

Self-Evaluation

Ask yourself some important questions. Write your answers in sentences for your teacher.

1. In this lesson, what part of your work was **excellent**?

2. In this lesson, what part of your work **needs improvement**?

3. If you want help for some of the work in this lesson, ask your teacher in this space.