Important Concepts...
Important Concepts of Grade 8 Mathematics

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Materials Required
Protractor
Ruler
Calculator

No Textbook Required

This is a stand-alone course.

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Publisher: Alberta Distance Learning Centre
Written by: Monica dHamrait
Reviewed by: Patty Rogerson

Project Coordinator: Donna Silgard
Preview/Review Publishing Coordinating Team:
Heather Martel and Nicole Mckeand

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Preview/Review Concepts for Grade Eight Mathematics

Teacher Key

W1 - Lesson 3:
Multiplying and Dividing Fractions
OBJECTIVES

By the end of this lesson, you will be able to:

• Express a given positive mixed number as an improper fraction and a given positive improper fraction as a mixed number
• Model the process of multiplying fractions using models and numerically
• Model the process of dividing fractions using models and numerically
• Apply rules for multiplying and dividing positive fractions, including mixed numbers
• Solve an expression involving fractions and apply the order of operations
• Solve problems involving the multiplication and division of fractions and mixed numbers

GLOSSARY

Denominator – the bottom number of a fraction, the number of equal sized parts a whole has been divided into.

Factors – the numbers that are multiplied to give a product. A factor of a given number will divide into the given number with no remainder left over. For example, the 4 and 5 are factors of 20.

Greatest Common Factor (GCF) – the largest factor that is common to a set of numbers. Example, the GCF of 12 and 18 is 6 because 6 divides into 12 twice and into 18 three times.

Improper Fraction – a fraction in which the numerator that is greater than the denominator.

Mixed number – a number represented by a whole number and a fraction.

Multiple – a number that is the product of a natural number and another number. For example, the multiples of 5 are 5, 10, 15, 20, etc.

Numerator – the top number of a fraction, the number of pieces being considered.

Simplify – reducing a fraction into its lowest terms. This is done by dividing both the numerator and denominator by the GCF.
W1 – Lesson 3: Multiplying and Dividing Fractions

Materials required:

- Paper, Pencil, and Calculator

Review: Converting between Improper Fractions and Mixed Numbers

To change a mixed number into an improper fraction, multiply the denominator by the whole number and add the numerator. This value becomes the numerator in the improper fraction. The denominator in the mixed number is the same denominator you use in the improper fraction.

For example:

\[
\frac{9}{4} = 2 \text{ and there is one left over}
\]

\[
\frac{9}{4} = 2\frac{1}{4} \quad \text{This is the remainder.}
\]

First, multiply the denominator by the whole number

For example: \( \frac{2\frac{1}{3}}{3} = \frac{3 \times 2 + 1}{3} = \frac{7}{3} \)

Second, add the numerator.
**Part 1: Multiplying Fractions**

When you multiply fractions, what you are really doing is finding a fraction of a fraction. Fractions can be multiplied using the area model.

Use the area model to multiply \( \frac{2}{3} \times \frac{1}{5} \).

First use a square to represent \( \frac{2}{3} \) vertically.

Then, represent \( \frac{1}{5} \) horizontally on the same square.

The shaded squares that overlap represent the numerator of the answer. The total number of parts in the square represents the denominator.

In this case, the numerator would be 2, and the denominator would be equal to 15.

Thus, \( \frac{2}{3} \times \frac{1}{5} = \frac{2}{15} \).
When multiplying fractions numerically, remember these steps:

**Step 1:** Convert any mixed numbers into improper fractions.
**Step 2:** Simplify the numerators and denominators. To do this, divide the numerators and denominators by a common factor.
**Step 3:** Multiply the numerators.
**Step 4:** Multiply the denominators.
**Step 5:** If an improper fraction results, change it into a mixed number.

**Example 1**

\[
\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}
\]

**Example 2**

\[
2\frac{1}{3} \times 3 = \frac{7}{3} \times \frac{3}{4} = \frac{7 \times 3}{3 \times 4} = \frac{7}{4} = 1\frac{3}{4}
\]

Cancel out the 3's.
Example 3

\[
\frac{7}{8} \times 5 = \frac{7}{8} \times 5
\]

\[
= \frac{7 \times 5}{8 \times 1}
\]

\[
= \frac{35}{8}
\]

\[
= 4 \frac{3}{8}
\]

Example 4

\[
3 \frac{1}{3} \times 5 \frac{1}{4} = \frac{10}{3} \times \frac{21}{4}
\]

\[
= \frac{16 \times 21}{1 \times 2}
\]

\[
= \frac{35}{2}
\]

\[
= 17 \frac{1}{2}
\]

Practice Questions

1. \[
\frac{3}{5} \times 3 = \]

\[
= \frac{3 \times 3}{5 \times 1}
\]

\[
= \frac{9}{5}
\]

\[
= 1 \frac{4}{5}
\]
2. \[ \frac{5}{6} \times \frac{2}{3} = \]

\[ \frac{5 \times 2}{6 \times 3} = \frac{5 \times 1}{3 \times 3} = \frac{5}{9} \]

3. \[ \frac{5}{3} \times \frac{3}{5} = \]

\[ \frac{5 \times 3 \times 3 \times 5}{3 \times 5} = \frac{28}{5} \times \frac{3}{8} = \frac{7 \times 3}{5 \times 2} = \frac{21}{10} = \frac{2 \frac{1}{10}}{10} \]

4. \[ \frac{11}{2} \times \frac{2}{5} = \]

\[ \frac{11}{2} \times \frac{2}{5} = \frac{3 \times 12}{5} = \frac{3 \times 6}{1 \times 5} = \frac{18}{5} = \frac{3 \frac{3}{5}}{5} \]
Part 2: Dividing Fractions

When you divide 12 by 3, you are determining how many groups of 4 are in 12. When 12 is divided into groups of 4, the result is 3. Dividing fractions uses a similar method.

You can divide fractions using a number line.

Let’s use $\frac{1}{5} \div \frac{4}{20}$ as an example. When dividing 5 by $\frac{1}{4}$, you must determine how many groups of $\frac{1}{4}$ are in 5.

There are 20 groups of $\frac{1}{4}$ in 5.

Thus, $5 \div \frac{1}{4} = 20$.

When dividing fractions numerically, remember these steps:

**Step 1:** Convert any mixed numbers into improper fractions.

**Step 2:** Multiply the first term by the reciprocal of the second term. The reciprocal is the “flip” of the fraction or number.

**Step 3:** Multiply the numerators.

**Step 4:** Multiply the denominators.

**Step 5:** If an improper fraction results, change it into a mixed number.
Example 1

\[
\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}
\]

The reciprocal of 2 is \(\frac{1}{2}\)

Example 2

\[
12 \div \frac{2}{5} = \frac{12}{1} \times \frac{5}{2} = \frac{6 \times 5}{1 \times 1} = 30
\]

Multiply by the reciprocal of the second term

Example 3

\[
\frac{3}{7} \div \frac{1}{5} = \frac{3}{7} \times \frac{5}{1} = \frac{3 \times 5}{7 \times 1} = \frac{15}{7} = 2\frac{1}{7}
\]
Example 4

\[
2 \frac{4}{7} \div 3 \frac{3}{8} = \frac{18}{7} \div \frac{27}{8}
\]

\[
= \frac{18}{7} \times \frac{8}{27}
\]

\[
= \frac{216}{7} \times \frac{8}{27}
\]

\[
= \frac{2 \times 8}{7 \times 3}
\]

\[
= \frac{16}{21}
\]

Practice Questions

1. \[
8 \div \frac{2}{3} = \]

\[
8 \div \frac{2}{3} = \frac{8}{1} \times \frac{3}{2}
\]

\[
= \frac{4 \times 3}{1 \times 2}
\]

\[
= \frac{4 \times 3}{1 \times 1}
\]

\[
= 12
\]

2. \[
\frac{5}{12} \div 4 = \]

\[
\frac{5}{12} \div 4 = \frac{5}{12} \times \frac{1}{4}
\]

\[
= \frac{5 \times 1}{12 \times 4}
\]

\[
= \frac{5}{48}
\]
3. \( \frac{2}{7} + \frac{1}{5} = \) 

\[
\frac{2}{7} + \frac{1}{5} = \frac{2 \times 5}{7 \times 1} = \frac{2 \times 5}{7 \times 1} = \frac{10}{7} = 1 \frac{3}{7}
\]

4. \( \frac{2}{5} + 1\frac{3}{4} = \)

\[
\frac{2}{5} + 1\frac{3}{4} = \frac{12}{5} + \frac{4}{7}
\]

\[
= \frac{12 \times 4}{5 \times 7} = \frac{48}{35}
\]

\[
= 1 \frac{13}{35}
\]
Part 3: Applying Order of Operations

When calculating the answer to an expression with many mathematical operations you must follow the order of operations. Use the acronym BEDMAS to help you.

**Example 1**

Evaluate the following expression \( \frac{3}{4} + \left(\frac{1}{5}\right)^3 \div \frac{4}{5} \)

**Answer:**

**Step 1:** Since there are no brackets present, evaluate the exponent.

\[
\frac{3}{4} + \left(\frac{1}{5}\right)^3 \div \frac{4}{5} = \frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} = \frac{3}{4} + \frac{1}{125} \div \frac{4}{5}
\]
Step 2: Complete the division.

\[
\frac{3}{4} + \frac{1}{125} + \frac{4}{5} = \frac{3}{4} + \frac{1}{125} \times \frac{5}{4}
\]

\[
\frac{3}{4} + \frac{1}{125} \times \frac{5}{4}
\]

\[
\frac{3}{4} + \frac{1}{100}
\]

Step 3: Complete the addition.

\[
\frac{3}{4} + \frac{1}{100} = \frac{75}{100} + \frac{1}{100} = \frac{76}{100} = \frac{19}{25}
\]
Example 2

Evaluate the following expression \( \left( \frac{5}{7} + \frac{1}{2} \right) \left( \frac{2}{3} + \frac{1}{4} \right) \).

Answer:

Step 1: Solve the operations in the first set of brackets.

\[
\left( \frac{5}{7} + \frac{1}{2} \right) \left( \frac{2}{3} + \frac{1}{4} \right) \\
= \left( \frac{5}{7} \times \frac{2}{1} \right) \left( \frac{2}{3} + \frac{1}{4} \right) \\
= \left( \frac{10}{7} \right) \left( \frac{2}{3} + \frac{1}{4} \right)
\]

Step 2: Solve the operations in the second set of brackets.

\[
\left( \frac{10}{7} \right) \left( \frac{2}{3} + \frac{1}{4} \right) \\
= \left( \frac{10}{7} \right) \left( \frac{5}{3} + \frac{1}{4} \right) \\
= \left( \frac{10}{7} \right) \left( \frac{20}{12} + \frac{3}{12} \right) \\
= \left( \frac{10}{7} \right) \left( \frac{23}{12} \right)
\]

Step 3: Solve the operations in the second set of brackets.

\[
\left( \frac{10}{7} \right) \left( \frac{23}{12} \right) \\
= \left( \frac{5 \times 10}{7} \right) \left( \frac{23}{12} \right) \\
= \frac{5 \times 23}{7 \times 6} \\
= \frac{115}{42} \\
= \frac{2}{42} + \frac{31}{42}
\]
Practice Questions

1. \( \frac{1}{2} + \frac{1}{7} \times \frac{1}{2} = \)

\[
= \frac{1}{2} + \frac{1}{7} \times \frac{1}{2} \\
= \frac{1}{2} + \frac{1}{14} \\
= \frac{7}{14} + \frac{1}{14} \\
= \frac{8}{14} \\
= \frac{4}{7}
\]

2. \( \left( \frac{2}{3} \right)^2 \times \left( \frac{5}{6} + \frac{10}{9} \right) = \)

\[
= \left( \frac{2}{3} \right)^2 \times \left( \frac{5}{6} + \frac{10}{9} \right) \\
= \left( \frac{2}{3} \right)^2 \times \left( \frac{5}{6} \times \frac{9}{10} \right) \\
= \left( \frac{2}{3} \right)^2 \times \left( \frac{5}{6} \times \frac{9}{10} \right) \\
= \left( \frac{2}{3} \right)^2 \times \frac{3}{4} \\
= \frac{4}{9} \times \frac{3}{4} \\
= \frac{4}{3} \times \frac{3}{4} \\
= \frac{1}{3}
\]
3. \[
\frac{3}{8} + \left( \frac{2}{3} - \frac{1}{6} \right) \div \frac{2}{5} = \\
\frac{3}{8} + \left( \frac{4}{6} - \frac{1}{6} \right) \div \frac{2}{5} \\
= \frac{3}{8} + \frac{3}{6} \div \frac{2}{5} \\
= \frac{3}{8} + \frac{1}{2} \div \frac{5}{2} \\
= \frac{3}{8} + \frac{1}{2} \times \frac{2}{5} \\
= \frac{3}{8} + \frac{5}{8} \\
= \frac{8}{8} + \frac{5}{8} \\
= \frac{13}{8} \\
= 1 \frac{5}{8}
\]
Lesson 3: Assignment

1. \[ \frac{5}{6} \times \frac{1}{3} = \]
\[ \frac{5 \times 1}{6 \times 3} = \frac{5}{18} \]

2. \[ \frac{9}{10} \times \frac{1}{6} = \]
\[ \frac{9 \times 1}{10 \times 6} = \frac{3}{20} \]

3. \[ 4 \frac{4}{7} \times 2 = \]
\[ 4 \frac{4}{7} \times 2 = \frac{32}{7} \times 2 = \frac{32 \times 2}{7} = \frac{64}{7} = 9 \frac{1}{7} \]

4. \[ 2 \frac{4}{5} \times \frac{3}{8} = \]
\[ 2 \frac{4}{5} \times \frac{3}{8} = \frac{14}{5} \times \frac{3}{8} = \frac{42}{40} = \frac{21}{20} = 1 \frac{1}{20} \]

5. \[ 1 \frac{7}{8} \times 2 \frac{2}{5} = \]
\[ 1 \frac{7}{8} \times 2 \frac{2}{5} = \frac{15}{8} \times \frac{12}{5} = \frac{180}{40} = \frac{9}{2} = 4 \frac{1}{2} \]

6. \[ \frac{7}{12} \times \frac{3}{4} = \]
\[ \frac{7 \times 3}{12 \times 4} = \frac{7 \times 15}{12 \times 4} = \frac{105}{48} = \frac{35}{16} = 2 \frac{3}{16} \]
7. \( \frac{3}{5} \div \frac{1}{4} = \)

\[ \frac{3}{5} \div \frac{1}{4} = \frac{3}{5} \times \frac{4}{1} = \frac{12}{5} = 2 \frac{2}{5} \]

8. \( \frac{5}{6} \div \frac{2}{3} = \)

\[ \frac{5}{6} \div \frac{2}{3} = \frac{5 \times 3}{6 \times 2} = \frac{5}{4} = 1 \frac{1}{4} \]

9. \( 8 \div 1 \frac{1}{4} = \)

\[ 8 \div 1 \frac{1}{4} = 8 \div \frac{5}{4} = \frac{8 \times 4}{5} = \frac{32}{5} = 6 \frac{2}{5} \]

10. \( \frac{3}{5} \div \frac{1}{5} = \)

\[ \frac{3}{5} \div \frac{1}{5} = \frac{3 \times 1}{5} \div \frac{5}{5} = \frac{17}{1} = 17 \]

11. \( \frac{3}{8} \div 2 = \)

\[ \frac{3}{8} \div 2 = \frac{43}{8} \div 2 = \frac{43 \times 1}{16} = 2 \frac{11}{16} \]

12. \( 1 \frac{10}{11} \div 3 \frac{1}{2} = \)

\[ 1 \frac{10}{11} \div 3 \frac{1}{2} = \frac{21}{11} \div \frac{7}{2} = \frac{321}{11} \times \frac{2}{11} = \frac{6}{11} \]
13. \[2 \times \frac{1}{4} = \]
\[2 \times \frac{1}{4} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{2} \]

14. \[\frac{6}{13} \div 3 = \]
\[\frac{6}{13} \div 3 = \frac{2}{13} \times \frac{1}{3} = \frac{2}{39} \]

15. \[9 \div \frac{3}{11} = \]
\[9 \div \frac{3}{11} = 9 	imes \frac{11}{3} = 3 \times \frac{11}{3} = 33 \]

Solve the following problems involving fractions. All the answers must be in lowest terms, include the units, and written in complete sentences.

16. An elastic band will stretch to be \(4\frac{1}{8}\) times its original size. If the elastic back is \(10\frac{2}{3}\) cm long, to what length will it stretch?

The question requires you to multiply the length of the elastic band by the number of times it stretches.

\[4\frac{1}{8} \times 10\frac{2}{3} = \frac{33}{8} \times \frac{32}{3} \]
\[= \frac{1105}{18} \times \frac{32}{3} \]
\[= \frac{44}{1} \]
\[= 44 \]

The elastic band will stretch to a length of 44cm.
17. Captain Dianne is in her canoe and notices it has $20\frac{1}{2}$ L of water in it. She has a container that holds $3\frac{1}{3}$ L of water. How many scoops of water will must she make to bail all the water out of her canoe?

To determine how many scoops of water it will take to remove all the water, you must divide the volume of the water inside the boat by the volume of the container.

\[
\frac{20\frac{1}{2}}{3\frac{1}{3}} = \frac{41}{2} \div \frac{10}{3} = \frac{41}{2} \times \frac{3}{10} = \frac{123}{20} = 6\frac{3}{20}
\]

It will take $6\frac{3}{20}$ scoops of the container to bail out all the water.

Therefore, Dianne must make 7 scoops to get all the water out.

18. How many complete pieces of rope $\frac{8}{9}$ metres long can be cut from a roll of rope that is 36 metres long?

To determine how many smaller pieces of rope can be cut from the total rope, you must divide the total length of the rope by $\frac{8}{9}$.

\[
36 + \frac{8}{9} = \frac{36}{1} \times \frac{9}{8} = \frac{326}{1} \times \frac{9}{8} = \frac{81}{2} = 40\frac{1}{2}
\]

You can cut 40 complete pieces from the roll of rope.
19. Corbin and Sheila are in the widget-selling business together. Since Shelia invested more money into the business than Corbin did, she is entitled to \( \frac{3}{5} \) of the profits.

For every widget they sell, they earn $18.50 in profit. How much will Corbin get for each widget that is sold? Express the answer as a fraction and in dollars.

**Step 1: Determine how much of the profits Corbin will get.**

*If Sheila gets \( \frac{3}{5} \) of the profits then Corbin will get \( \frac{2}{5} \) of the profits.*

**Step 2: Calculate \( \frac{2}{5} \) of $18.50 by multiplying the two numbers.**

\[
\frac{2}{5} \times 18.50 \text{ or } \frac{2 \times 37}{5} \text{ or } 7.40
\]

Corbin will get $7 \( \frac{2}{5} \) or $7.40 for each widget sold.

20. \( \frac{1}{2} + \left( \frac{3}{4} - \frac{1}{8} \right) \) =

\[
= \frac{1}{2} + \left( \frac{3 \times 2}{4 \times 2} - \frac{1}{8} \right)
= \frac{1}{2} + \left( \frac{6}{8} - \frac{1}{8} \right)
= \frac{1}{2} + \frac{5}{8}
= \frac{4 + 5}{8}
= \frac{9}{8}
= 1 \frac{1}{8}
\]
21. \[
\frac{2}{3} \times \left( \frac{1}{2} \right)^3 = \\
\frac{2}{3} \times \frac{1^3}{2^3} \\
\frac{2}{3} \times \frac{1}{8} \\
\frac{1}{3} \times \frac{1}{8} \\
\frac{1}{12}
\]

22. \[
11 - 5 + \frac{2}{3} = \\
11 - 5 + \frac{2}{3} \\
11 - \frac{5 \times 3}{1 \times 2} \\
11 - \frac{15}{2} \\
\frac{22}{2} - \frac{15}{2} \\
\frac{7}{2} \\
3 \frac{1}{2}
\]
23. \[
\left( \frac{3}{7} + \frac{2}{3} \right) \left( \frac{9}{10} - \frac{3}{5} \right) =
\]
\[
\left( \frac{3}{7} + \frac{8}{3} \right) \left( \frac{9}{10} - \frac{3}{5} \right)
\]
\[
\left( \frac{9}{21} + \frac{56}{21} \right) \left( \frac{9}{10} - \frac{3}{5} \right)
\]
\[
\left( \frac{65}{21} \right) \left( \frac{9}{10} - \frac{3}{5} \right)
\]
\[
\left( \frac{65}{21} \right) \left( \frac{9}{10} - \frac{6}{10} \right)
\]
\[
\left( \frac{65}{21} \right) \left( \frac{3}{10} \right)
\]
\[
\left( \frac{\frac{65}{21}}{\frac{3}{10}} \right)
\]
\[
\frac{13}{14}
\]
24. \[
\left( \frac{8 \times 3 + 5}{9 \times 4 + 6} \right) \div 3 \frac{3}{5} = \\
= \left( \frac{8 \times 3 + 5}{9 \times 4 + 6} \right) \div 3 \frac{3}{5} \\
= \left( \frac{2 \times 8 + 5}{3 \times 9 + 6} \right) + 3 \frac{3}{5} \\
= \left( \frac{2 + 5}{3 + 6} \right) + 3 \frac{3}{5} \\
= \left( \frac{3}{6} \right) + 3 \frac{3}{5} \\
= \frac{9 + 3 \frac{3}{5}}{6} \\
= \frac{9 + 18}{6} \\
= \frac{27}{18} \\
= \frac{5}{12}
\]